FEM calculations for the glass beam under various loads

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Keywords

1=Glass beam 2=Finite elements 3=Numerical modeling

Abstract

This paper focuses on the glass beam behavior under the influence of external forces. The rib strength characteristics under different combinations of fastening, support and load are analyzed. Beams fasten in the ends or to the cross-sectional perimeters while load changes from uniform to the spot concentrated force. A comparison of structural strengths for the constructions with same shape but with different sizes and number of the details is made. The results were obtained by the use of the program developed by authors on the basis of the finite element method. Today we are successfully using this new program for the calculation of glass beams and stiffening ribs.

Introduction

The main aim of this work is to research glass beam behavior under the influence of external forces. Nowadays the glass is widely using not only for facades decoration and building interiors but also it is a base material of supporting constructors. There are special mathematical programs that design the behavior of structures made of different materials under the impact of various external forces.

Figure 1.

Figure 2.

In today's world the modeling of various designs is increasingly uses the finite element method. The main advantages of this method are the flexibility and variety of meshes, the simplicity includes of natural boundary conditions and the presence of standard methods for constructing discrete problem for random fields. Also this method provides more accurate results not only by mesh refinement but also by increasing the element's orders.

The aim of this work is the development of the program, easy to use, based on the finite element method, and modeling of the glass rib strength with this program. The Glass Research Institute already made a lot of work in the development of models for numerical analysis of glass construction behavior, but now we started our research of finite element methods possibilities.

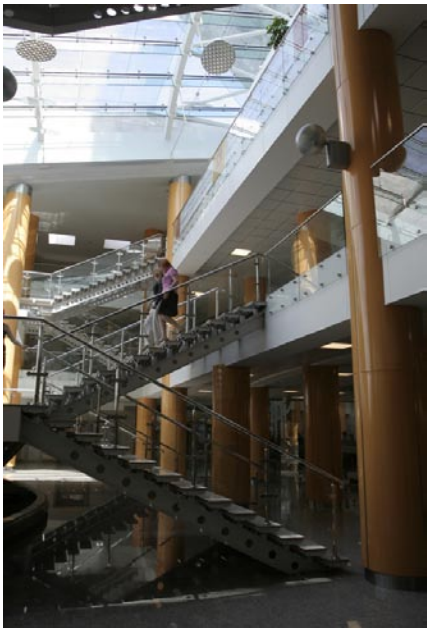

Figure 3. **Figure 4.**

Problem definition

The equation for bending beam is as follows:

$$\frac{d^2}{dx^2}\left(h(x)\frac{d^2u}{dx^2}(x)\right) = f(x) \quad \forall x \in \Omega. \tag{1}$$

Here Ω is the region representing the beam, $h(x)$ – the function that presents the product of elastic modulus and the moment of inertia, $f(x)$ – the external load function.

In case of clamped beam the boundary conditions represented the deflections and slopes:

$$\begin{aligned} u_a &= u(a), \quad u_b = u(b), \\ du_a &= u'(a), \quad du_b = u'(b). \end{aligned} \tag{2}$$

So-called weak formulation reads as follows: find the solution of integral equation

$$\int_\Omega h\Delta u \Delta v dx = \int_\Omega f v dx. \quad , \tag{3}$$

where $v(x)$ is sufficiently regular test function of the choice of which solution $u(x)$ is independent, and both functions $u(x)$ and $v(x)$ belong to a certain functional space $V \in H^2(\Omega)$ of functions which satisfy the boundary conditions.

Boundary fixed beam

Let's consider the glass beams with a rectangular cross-section. In the case of clamped ends the boundary conditions are given by:

$$\begin{aligned} u_a &= 0, \quad u_b = 0, \\ du_a &= 0, \quad du_b = 0, \end{aligned} \tag{4}$$

and a space in which we will seek a solution transforms to

$$V = \{v \in H^2(\Omega); u(a) = u(b) = \nabla u(a) = \nabla u(b) = 0\}. \tag{5}$$

For the beam size of 4000 × 300 × 15 mm and force 440 N the deflection value at the uniform and spot concentrated loads are shown in Figure 5 (blue - with a uniform load, a red - with a spot concentrated force).

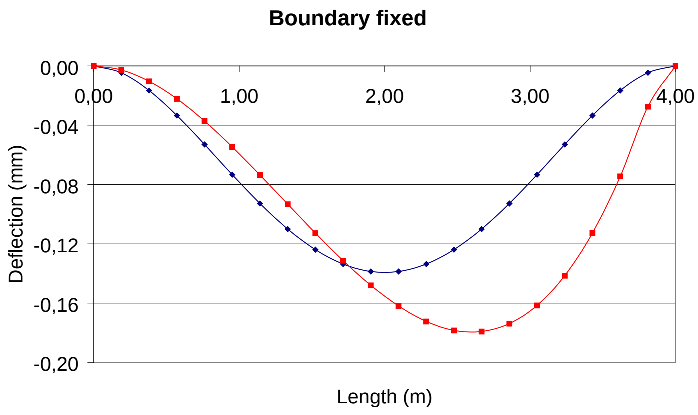

Figure 5.

Graphs show that beam under point load deflects more than under the uniform, so the constructions with only uniform load are more durable then with point load. Note, that uniform load and concentrated load here are equivalent in terms of total force. The point of force application in the model can be moved freely, so figure 5 shows shifted point load.

Point fixed beam

For triple glass of retaining wall calculations can be made not for plates, but as for beams. In this case, the mathematical model is as follows: equation (3) remains unchanged, and the boundary conditions are replaced by the following:

$$u_{a'} = u(a), \quad u_{b'} = u(b),$$
$$du_{a'} = u'(a), \quad du_{b'} = u'(b). \tag{6}$$

Here a' and b' are not the ends of the beam interval, but the coordinates of the fixed points. Figure 6 shows the deflection of point fixed beam with uniform and spot concentrated loads (blue - with a uniform load, a red - with the spot concentrated one).

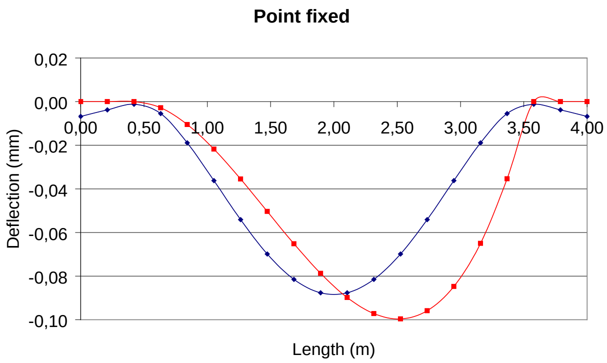

Figure 6.

And again, the point loaded beam deflects more than the beam under uniform load.

Composite beam

Now let’s consider the model of composite beam. For example, we have one beam size of 4000×300×45 mm and three beams size 4000×300×15 mm. Compile one composite beam size of 4000×300×45 mm from the last three beams. Figure 7 depicts a deflection diagrams for this model (depicted in red composite beam deflection, in blue – whole beam deflection).

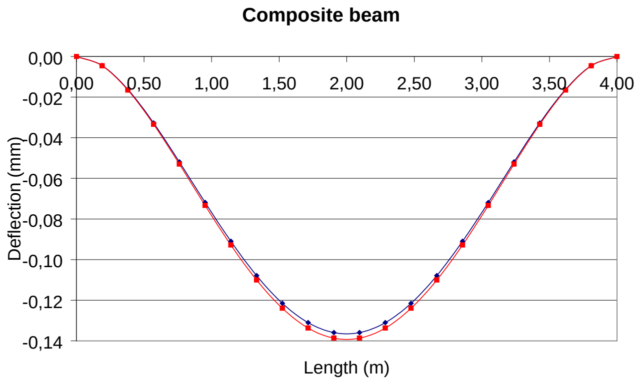

Figure 7.

The composite beam deflection for this model is 1.9% more than a deflection of whole beam, which allows using composite beams instead of expensive whole thick beams.

Conclusion

The most important results of this work are the following:

- the program for design of the behavior of glass rib strength by means of finite element method is developed;
- modification of existing algorithms for the enumeration of elements in solving the problem of the deflection of the beam are made;
- the calculations of test tasks is made. Within the limits of error obtained numerical solutions are conforming with the analytical solutions;
- comparing the results of calculations with the published results of other authors is made;
- the dependence of the accuracy of the obtained solutions from order entry is conducted. With increasing of the order of elements the accuracy of the method increases;
- the deflection of glass beam with various fastening and various external forces are calculated.

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